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IS FACTORIZATION FOR ISOLATED PHOTON CROSS SECTIONS BROKEN?

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Abstract

Recently the factorization property has been claimed to be broken in the cross section for the production of isolated prompt photons emitted in the final state of hadronic e^+e^- annihilation. We contest this claim.

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The authors of [1] claim to have found a breakdown of factorization in the cross section for the production of isolated prompt photons emitted in the final state of hadronic e^+e^- annihilation. A photon is said to be isolated if it is accompanied by less than a specified amount of hadronic energy, e.g. $E_h^{cone} \leq \epsilon_h E_\gamma$, in a cone of half angle δ about the direction of the photon momentum.

This claim is based on a partial calculation of the process $\gamma^* \rightarrow g\bar{q}q\gamma$ (and associated virtual gluon corrections) at $\mathcal{O}(\alpha_s\alpha_f)$ in the kinematical configuration for which the emitted photon is collinear to the fragmenting quark. This contribution (using dimensionnal regularization with $d = 4 - 2\epsilon$ for long distance singularities) takes the following form :

$$\frac{d\sigma}{dx_\gamma} = \int_{\max(x_\gamma, x_\gamma^c)}^1 \frac{dz}{z} \left[-\frac{1}{\epsilon} \frac{\alpha_{em}}{2\pi} P_{\gamma q}^{(0)}(z) \right] \left[\frac{d\hat{\sigma}^{\gamma^* \rightarrow (g)\bar{q}q}}{dx_1}(x_1) \right]_{x_1 = \frac{x_\gamma}{z}} \quad (1)$$

where $x_\gamma = 2E_\gamma/\sqrt{s}$ is the energy of the emitted photon scaled by the c.m.s. total energy and $x_\gamma^c = 1/(1 + \epsilon_h)$. The first factor contains the collinear singularity due to the splitting $q \rightarrow q\gamma$. The authors of [1] focus on the other factor - the residue of the collinear pole $q \rightarrow q\gamma$ - which is expected to be the cross section of the short distance subprocess $\gamma^* \rightarrow (g)\bar{q}q$. They find that this quantity is actually plagued with infrared (IR) $1/\epsilon$ singularities surviving from an incomplete cancellation between real and virtual gluon contributions. Relying on this, they conclude that “the conventional factorization theorem for the cross section of isolated photons in e^+e^- annihilation breaks down when $x_\gamma \sim 1/(1 + \epsilon_h)$ ”, so that “the cross section cannot be factored into a sum of terms each having the form of an infrared-safe partonic hard part times a corresponding parton-to-photon fragmentation function”.

We contest this conclusion, and argue that

- (a) the IR $1/\epsilon$ singularities on which the claim relies are actually irrelevant
- (b) on the other hand, the appearance of accompanying large IR logarithms when $x_\gamma \sim 1/(1 + \epsilon_h)$ is the relevant point to be discussed, as it is generally the case when the phase space available for gluon emission is restricted. However it does not necessarily mean that the cross section is not factorizable.

Dropping inessential terms, the expression for $\frac{d\hat{\sigma}^{\gamma^* \rightarrow (g)\bar{q}q}}{dx_1}$ takes the following form :

$$\frac{d\hat{\sigma}^{\gamma^* \rightarrow (g)\bar{q}q}}{dx_1} = \frac{d\hat{\sigma}_{virtual}^{\gamma^* \rightarrow \bar{q}q}}{dx_1} \theta(x_\gamma - x_\gamma^c) + \int |M^{\gamma^* \rightarrow g\bar{q}q}|^2 [dPS^{(3)}] \Theta_{iso} \quad (2)$$

$dPS^{(3)}$ is the three particle phase space element :

$$dPS^{(3)} \propto dy_{13} dy_{23} dy_{12} (y_{13} y_{23} y_{12})^{-\epsilon} \delta(1 - y_{13} - y_{23} - y_{12}) \delta(1 - y_{23} - x_1) \quad (3)$$

where the y_{ij} 's are the scaled invariant masses s_{ij}/s with indices 1, 2, 3 referring to the fragmenting quark, the anti-quark and the gluon, respectively. The symbol Θ_{iso} stands for the phase space restrictions imposed by the isolation about the photon: either the gluon and the antiquark are outside the cone, however the fragmented quark collinear to the photon has to be not too hard, or the gluon (resp. the antiquark) is inside the cone and not too hard, this being possible only if the photon is energetic enough. For the sake of simplicity, in the transition matrix element squared

$$|M^{\gamma^* \rightarrow g\bar{q}q}|^2 \propto (1 - \epsilon) \left(\frac{y_{13}}{1 - x_1} + \frac{1 - x_1}{y_{13}} \right) + \frac{2}{1 - x_1} \frac{y_{12}}{y_{13}}, \quad (4)$$

we keep only the terms which may produce the IR singularities at $x_1 \sim 1$ discussed by [1]. Three cases can be distinguished:

When $x_\gamma < x_\gamma^c$, Θ_{iso} can be written

$$\Theta_{iso} = \theta(x_\gamma^c - x_\gamma) \theta(z - x_\gamma^c) \theta(y_\delta - y_{13}) \theta(y_\delta - y_{12}) \quad (5)$$

$$\text{with } y_\delta = x_1 \frac{(1 - x_1) \sin^2 \frac{\delta}{2}}{1 - x_1 \sin^2 \frac{\delta}{2}}, \quad y_\epsilon = \left(\frac{x_\gamma}{x_\gamma^c} - 1 \right), \quad y_m = \min(y_\delta, y_\epsilon). \quad (6)$$

Necessarily $z \geq x_\gamma^c$ so that $x_1 \leq x_\gamma/x_\gamma^c < 1$. Hence $\frac{d\hat{\sigma}^{\gamma^* \rightarrow (g)\bar{q}q}}{dx_1}$ is free from any IR $1/\epsilon$ poles in the range $x_\gamma < x_\gamma^c$, a conclusion on which we agree with [1].

When $x_\gamma > x_\gamma^c$, Θ_{iso} can be written as:

$$\Theta_{iso} = \theta(x_\gamma - x_\gamma^c) + \theta(x_\gamma - x_\gamma^c) [\theta(y_\delta - y_{13}) \theta(y_\delta - y_{12}) - \theta(y_m - y_{13}) \theta(y_m - y_{12})] \quad (7)$$

The first term of Θ_{iso} combined with the virtual part exactly reconstructs the fully inclusive case, so that, if there are extra IR singularities, they will come from the other two terms on which we now focus our attention. The variables y_m and y_δ coincide over a finite neighbourhood $x_1^{lim} \leq x_1 \leq 1$ of $x_1 = 1$ (x_1^{lim} is determined by the condition $y_\delta = y_\epsilon$, and is $\sim 1 - y_\epsilon \cot^2 \frac{\delta}{2}$ for x_γ close enough to x_γ^c .) For this reason, the “ y_m ” and “ y_δ ” contributions of Θ_{iso} cancel against each other in this neighbourhood, thus preventing the appearance of any IR $1/\epsilon$ pole.

The only tricky point is at precisely $x_\gamma = x_\gamma^c$. Then y_m vanishes, the gluon and the antiquark must be both outside the cone. After integration over the y_{ij} 's and addition of the virtual contribution, one gets:

$$\begin{aligned} \frac{d\hat{\sigma}^{\gamma^* \rightarrow (g)\bar{q}q}}{dx_1}(x_1) &\propto (1-x_1)^{-1-\epsilon} \left[\left(\frac{2}{\epsilon} \right) \left((y_\delta)^{-\epsilon} - 1 \right) - \frac{3}{2} \right] - \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} \right) \delta(1-x_1) \\ &\sim - \left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left(\frac{3}{2} + \ln \cot^2 \frac{\delta}{2} \right) \right] \delta(1-x_1) + (finite) \end{aligned} \quad (8)$$

which, after integration over z , i.e. over x_1 , apparently induces the appearance of $1/\epsilon$ IR poles in $\frac{d\sigma}{dx_\gamma}$ at $x_\gamma = x_\gamma^c$.

However one has to realize that these IR $1/\epsilon$ poles appearing only at $x_\gamma = x_\gamma^c$ in the calculation of $\frac{d\sigma}{dx_\gamma}$ are irrelevant both mathematically and physically for the following reasons :

from a mathematical point of view, $\frac{d\sigma}{dx_\gamma}$ is not an ordinary function with a point-wise meaning : it is a distribution, about which only smearings on smooth enough test functions are meaningful. For the moment we forget about the accompanying IR sensitive logarithms, which are discussed below. These IR $1/\epsilon$ poles are of zero measure : they do not exist over a finite range in x_γ but stand only *precisely* at $x_\gamma = x_\gamma^c$ and they are not weighted by any $\delta(x_\gamma - x_\gamma^c)$ or so. Hence any smearing washes out these $1/\epsilon$ poles, no matter how sharp but finite the smearing provided by the test function is;

from a physical point of view, such a smearing is understood in terms of some finite energy resolution which unavoidably happens (even using an ideal apparatus) in a measurement process always occurring during a finite time, according to the Heisenberg time-energy indetermination principle.

For these reasons, these spurious IR $1/\epsilon$ poles are not to be considered as evidence for any breakdown of factorization since they give no contribution to the observable.

The relevant point concerning the issue of factorization, and the question whether it is possible to define universal, transportable quantities such as fragmentation functions of partons into a photon isolated from its hadronic environment, is actually not discussed in [1]. Indeed accompanying IR logarithms appear due to the isolation criterion imposed about the photon, as it is generally the case when the phase space available for real gluon emission is restricted. These IR logarithms, which have to be taken proper care of, make the computed cross section semi-inclusive and IR sensitive, and ensure that the sole factorization of collinear singularities is

not enough, in other words, the long distance factor of the cross section is not simply governed by Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equations.

However their appearance does not necessarily mean that the cross section is not factorizable. As a simpler illustrative example, if instead of using an isolation criterion in terms of cone and energy, one considers that a parton is accompanying the photon if its relative transverse momentum with respect to the photon is less than some p_{max}^2 , the same analysis which leads to the DDT formula [2] allows to see that the IR logarithms exponentiate in a form factor of Sudakov type $\exp \left[-S \left(p_{\perp \gamma}^2, p_{max}^2 \right) \right]$ where $S \left(p_{\perp \gamma}^2, p_{max}^2 \right) \propto \alpha_s \ln^2 \left(p_{\perp \gamma}^2 / p_{max}^2 \right)$ (resulting from the incomplete cancellation of IR sensitivity between the deepest rung of the ladder whose largest scale is $\sim p_{max}^2$ and the hard subprocess whose typical scale is $\sim p_{\perp \gamma}^2$), so that in this case the cross section does take a factorized form. On the other hand, this means that the ansatz proposed in Ref. [3] to define the fragmentation function of a parton into an isolated photon is incorrect. It was defined there as the inclusive fragmentation function of this parton minus a fragmentation contribution into a photon accompanied by a collinear jet. The latter was claimed to be simply given by the same inclusive fragmentation function where the fragmentation scale would be fixed to $\sim p_{\perp}^2 \delta$ in the case of a cone criterion, or $\sim p_{max}$ in the case of our above-mentioned p_{max} toy model criterion. In the last case, one sees that the erroneous ansatz of [3] for the subtracted, accompanied contribution

$$D_{\gamma}^{accompanied} = D_{\gamma}^{inclusive}(z, p_{max}^2) \quad (9)$$

has to be corrected into

$$D_{\gamma}^{accompanied} = D_{\gamma}^{inclusive}(z, p_{max}^2) \exp \left[-S \left(p_{\perp \gamma}^2, p_{max}^2 \right) \right] \quad (10)$$

In the present case of isolation in terms of cone and energy, the IR logarithms are of the form $\ln^2 \left[\left(x_{\gamma} / x_{\gamma}^c - 1 \right) \cot^2 \delta / 2 \right]$ when $x_{\gamma} > x_{\gamma}^c$ or $\ln \left[1 - x_{\gamma} / x_{\gamma}^c \right] \ln \left[\left(1 - x_{\gamma} / x_{\gamma}^c \right) \tan^4 \delta / 2 \right]$ when $x_{\gamma} < x_{\gamma}^c$. They become large in the neighbourhood of $x_{\gamma} \sim x_{\gamma}^c$: from a conceptual point of view one has to study whether they can be factorized since they reflect long distance effects, and from a computational point of view one has to study whether they can be resummed since they destroy the relevance of the perturbative expansion at least in the neighbourhood of x_{γ}^c .

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